$$s = \left[\frac{a-b}{2(a+b)}(2r-1)+\tfrac{1}{2}\right];\ [\alpha] \equiv \text{greatest integer in } \alpha.$$

The expression is an integer only when $\xi_{2s} = \frac{1}{2}$ which does not occur in this case.

For the integral we have

$$\sin \pi \left\{\frac{b(2r-1)}{2(a+b)} + (r-1) + \left[\frac{a-b}{2(a+b)}(2r-1)+\tfrac{1}{2}\right]\right\} = \sin \pi(L + [M]) \text{ say.}$$

Now $L + M = 2r - 1$ and M is not an integer. Hence $(L + [M]) = 2r - 2$ and the sine in question enters the integral positively. The set ξ_{1r} contains all odd multiples of $(a + b)/2$ and b is prime to $(a + b)$ and odd. The angles are then all odd multiples of $\pi/(a + b)$ and all being distinct and in the first two quadrants must be precisely the set

$$\pi k/(a+b), \quad k = 1, 3, 5, \cdots (a+b) - 1.$$

The sum of this set of sines contributes to the integral

$$\frac{2}{\pi ab}\left(\frac{a}{\sin \pi/(a+b)}\right).$$

A similar count for the set $\sin 2\pi a\xi_{1r}$, using $L' - [M]$ instead of $L + [M]$, adds a term of the same form, a and b being interchanged.

Similarly the roots of the ξ_{2s} set with a contribute four times the sum of sines of all odd multiples less than the $(a - b)$th of $\pi/(a - b)$. With b the same set occurs with negative sign.

A similar method gives the formula in case 2.

350. Proposed by R. P. BAKER, University of Iowa.

Find a general formula for $d^n y/dx^n$ in terms of $d^k y/dt^k$ and $d^k x/dt^k$.

SOLUTION BY J. W. CLAWSON Collegeville, Pa.

Let δx, δy be increments of x, y when t takes the increment δt. Then, by Taylor's Theorem,

$$(1) \qquad \delta y = \delta x \frac{dy}{dx} + \frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2y}{dx^2} + \frac{\overline{\delta x}^3}{\underline{|3}}\frac{d^3y}{dx^3} + \cdots,$$

$$(2) \qquad \delta y = \delta \ \frac{dy}{dt} + \frac{\overline{\delta t}^2}{\underline{|2}}\frac{d^2y}{dt^2} + \frac{\overline{\delta t}^3}{\underline{|3}}\frac{d^3y}{dt^3} + \cdots,$$

$$(3) \qquad \delta t = \delta x \frac{dt}{dx} + \frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2t}{dx^2} + \frac{\overline{\delta x}^3}{\underline{|3}}\frac{d^3t}{dx^3} + \cdots.$$

From (2) and (3) we get

$$(4) \quad \delta y = \left(\delta x\frac{dt}{dx} + \frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2t}{dx^2} + \cdots\right)\frac{dy}{dt} + \left(\delta x\frac{dt}{dx} + \frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2t}{dx^2} + \cdots\right)^2\frac{1}{\underline{|2}}\frac{d^2y}{dt^2} + \cdots.$$

Comparing (1) and (4) and equating coefficients of same powers of δx we get

$$\frac{1}{\underline{|n}}\frac{d^n y}{dx^n} = \sum_{k=1}^{h=n} C_k \frac{1}{\underline{|k}}\frac{d^k y}{dt^k},$$

where C_k is the coefficient of $\overline{\delta x}^n$ in the expansion of

$$\left[\delta x\frac{dt}{dx} + \frac{\overline{\delta x}^2}{\underline{|2}}\frac{d^2t}{dx^2} + \cdots\right]^k.$$

This formula gives d^ny/dx^n in terms of d^ky/dt^k and d^kt/dx^k, which is not quite the formula asked for. Compare Greenhill's "D fferential and Integral Calculus," p. 180.

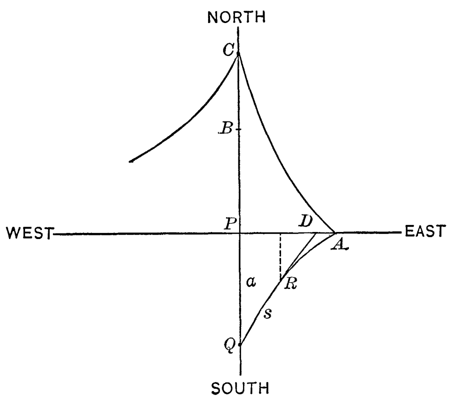
352. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

At point P there are n foxes. At Q, a rods south of P, there is a dog. The dog and the foxes are freed at the same instant and run at uniform speeds. Some of the foxes run east, some north, some west and some south. The dog runs first towards the foxes that ran east and always points toward them. He captures one of them and then instantly pursues the pack that ran north. In like manner, when he has captured one of them, he pursues those that ran west, then those that ran south, and then begins over again by pursuing the ones running east. If r is the ratio of the dog's speed to that of a fox, what is the total length of the n curves of pursuit.

(Generalization of a problem published in 1859 in the *Mathematical Monthly.*)

Solution by J. W. Clawson, Collegeville, Pa.

Let S be the position of a fox at any time after the chase began and R the corresponding position of the dog. Taking PA for the x-axis and PQ for the y-axis we have



$$QR = r\cdot PD, \quad \text{i. e.,} \quad s = r\left(x - \frac{y}{\frac{dy}{dx}}\right) \quad \text{or} \quad \frac{ds}{dx} = r\left[1 - \frac{\left(\frac{dy}{dx}\right)^2 - y\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}\right].$$

Hence,

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = ry\frac{d^2y}{dx^2}\Big/\left(\frac{dy}{dx}\right)^2.$$